

FRACTALS

NATURE'S SECRET CODE

Exploring self-similarity, chaos, and the architecture of nature



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Fractals: Nature's Secret Code

A mathematical journey between the order of chaos and the hidden geometries of the universe

by Michele Valentino

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Table of Contents

Preface	7
Chapter 1 - Beyond Euclid: The Birth of Roughness	9
Chapter 2 - The Hausdorff Dimension: Measuring Irregularity	15
Chapter 3 - Self-Similarity: The Mirror of Infinity	23
Chapter 4 - The Recursive Algorithm	29
Chapter 5 - The Pioneers: From Cantor to Peano	35
Chapter 6 - Chaos Theory: Order in Disorder	41
Chapter 7 - The Butterfly Effect and Dynamic Systems	49
Chapter 8 - Richardson and the Measurement of Coastlines	57
Chapter 9 - Chance and Statistics in Fractals	63
Chapter 10 - Mandelbrot at IBM: Technology Meets Theory	71
Chapter 11 - The Mandelbrot Set: The 'Map' of Fractals	77
Chapter 12 - Julia Sets: Dances in the Complex Plane	85
Chapter 13 - Linear vs. Non-Linear Fractals	91
Chapter 14 - Iteration of Powers of Z: Beyond the Square	97
Chapter 15 - Fractals in 3D: Mandelbulb and Mandelbox	101
Chapter 16 - The Escape Method: Visualizing the Invisible	113
Chapter 17 - Fractals and Nature: From Mountains to Lungs	119
Chapter 18 - Programming Fractals: A Technical Guide	123
Chapter 19 - Fractal Finance: Predicting the Unpredictable	127
Chapter 20 - Fractal Cryptography and the Future	133
Chapter 21 - Synthesis and Overview	139
Chapter 22 - Iterated Function Systems (IFS): Modeling Life	151
Chapter 23 - Beyond Determinism: Fractals and Quantum Mechanics	157
Chapter 24 - Frontiers of Chaos: Open Questions	161
Chapter 25 - Mandelbrot's Legacy: Conclusion and New Horizons	169
Illustrated Plates	176
About the Author	189
Acknowledgments	191

Preface

Returning to the world of fractals, thirty-five years after the publication of my essay *Viaggio nell'Universo Frattale* (Journey into the Fractal Universe), feels like reopening a dialogue with a part of myself that never stopped observing the world through the lens of complexity.

In 1991, that first book was born from the desire to recount the mathematical wonder hidden in the forms of nature — a wonder that then seemed confined to laboratories and images generated by the first computers. Today, that same geometry has become a language, an aesthetic, even a philosophy.

Over the decades, my professional life has intersected with finance, risk psychology, and technology. But fractals have remained a constant: a way to understand how order can emerge from chaos, how beauty can be born from repetition and infinite variation.

Fractals: The Secret Code of Nature is not just a return, but an evolution.

It is a visual and conceptual journey that unites science and art, numbers and emotion, rigor and wonder. Every image, every reflection, is an invitation to recognize the hidden structure that binds the microcosm to the macrocosm, the human mind to the universe.

After thirty-five years, the fascination of fractals has not diminished: it has been transformed.

Today I see them not only as formulas, but as metaphors for life itself — for its unpredictability, its secret harmony, its capacity to repeat itself without ever being the same. I could say that this book is born from the same curiosity that accompanied me then, but today that curiosity has new tools, new questions, and new images. In these years I have learned that complexity is not only a mathematical concept: it is a key to reading markets, human relationships, living systems.

For this reason, I have chosen to accompany theoretical reflection with a broader visual dimension, capable of showing what words alone cannot tell. The images that make up this volume are not simple illustrations: they are maps of the invisible, traces of a deeper order that runs through nature and the mind. It is in this dialogue between science and perception, between rigor and imagination, that I found the sense of returning to fractals after so many years.

When I was a boy, the first insights into artificial intelligence did not come from a computer science manual, but from the pages of the novels I read voraciously. **Asimov** (1920–1992), more than any other, had the rare ability to transform scientific speculation into a coherent vision of the future: robots capable of dialoguing with humans, logical systems able to make

autonomous decisions, machines that were not simple tools, but interlocutors. At the time, those ideas belonged to the territory of fiction; they were fascinating but distant hypotheses.

Today, as I write this book, I find myself interacting daily with an artificial intelligence that understands natural language, processes complex concepts, and provides answers with a naturalness that would have amazed even the most optimistic reader of the eighties. It is a historical transition that deserves recognition: what was once literary imagination has become an integral part of the scientific method and the creative process. It is not only technological progress, but a profound cultural change, which redefines the relationship between knowledge, tools, and human thought.

In this sense, Asimov is not just a literary reference of my youth: he is the ideal bridge between the science fiction that formed me and the reality with which I dialogue today while exploring the hidden geometry of nature.

May this reading open a door — small or large — onto a new way of observing the world.

Chapter 1 - Beyond Euclid: The Birth of Roughness

«The order of nature is born from the disorder that we cannot measure.»

«What appears to be chaos is simply a geometry we have not yet understood.»

Take a ruler and measure the coast of Norway. Now use a shorter one: the length will increase. Reduce the unit of measurement again, and the length increases yet again. Push this process to its logical extreme and you will find yourself before a coast of tendentially infinite length. This is not a measurement error: it is nature revealing the limits of the tools with which, for centuries, we have described the world.

This observation, known as the **coastline paradox**, was one of the starting points for Benoît **Mandelbrot (1924–2010)** when, in the sixties and seventies, he began to build a completely new mathematical system. Mandelbrot was not an academic mathematician in the traditional sense: he worked at the IBM Research Center in Yorktown Heights, where he analyzed noise in data transmission signals. Yet, observing those graphs full of irregular fluctuations, he saw what his colleagues did not see: **a structure that repeated itself at different scales**, obstinate and precise.

To understand the scope of this intuition, one must first understand what he was calling into question.

The geometry that taught us to see the world

For over two thousand years, Euclidean geometry was the language with which human beings described and built the world. In the *Elements*, Euclid defined perfect objects: points, lines, planes, circles, spheres, cones, cylinders. All shared one fundamental property: **smoothness**. A sphere is perfectly round, a line has no thickness, a circle has an exact circumference at any scale.

This geometry worked extraordinarily well for buildings, machines, trajectories, and orbits. The problem is that the real world **is not made of smooth shapes**.

As **Mandelbrot** recalled: *«Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line.»*

Nature is **irregular at all scales**. And for centuries we did not have a mathematics capable of describing this irregularity.

Mandelbrot at IBM: when noise has a structure

In the sixties, IBM engineers were struggling with signals disturbed by apparently random errors. Classical statistical models failed to predict them: it was assumed they were white noise, devoid of memory.

Mandelbrot observed the same data and saw something else. He noticed that error-free periods were followed by clusters of errors, and that these clusters repeated themselves at different time scales. Zooming in on any portion of the graph, the structure remained statistically similar to the whole. It was **self-similarity**.

The same pattern appeared in historical cotton price data: daily fluctuations had the same statistical structure as monthly and annual ones.

Very distant phenomena — electrical signals, financial markets, coastlines, mountains — showed the same property: **a recurring geometry, not smooth, not random**.

Mandelbrot had found a common thread. Now a mathematical language was needed to describe it.

Smooth versus rough

A Euclidean curve, enlarged, becomes simpler and simpler: at sufficiently small scales it is indistinguishable from a straight line. It is **differentiable**, its local complexity is zero.

A fractal object behaves in the opposite way: the more you enlarge it, the more details emerge. The coast of Norway, from a satellite, shows fjords; from an airplane, inlets; from the ground, rocks and cracks; under a microscope, further structures. The detail never disappears.

This property is called **self-similarity**: every part, at any scale, resembles the whole, in an identical or statistical way.

The difference is not aesthetic: it is mathematical. A smooth curve has a well-defined length. A fractal curve can have a length that grows without limit as the unit of measurement decreases.

The term that changed everything

In 1975, **Mandelbrot** coined the term **fractal** (*fractus*, “broken”) to describe objects whose structure is fragmented and whose **dimension is not an integer**.

A curve may have dimension 1.26; a surface, 2.7. These values measure how much an object “fills” space in an irregular way. The formal concept — the **Hausdorff dimension** — will be addressed in the next chapter.

Non-Euclidean geometries and new metrics

Fractal geometry does not belong to the tradition of nineteenth-century non-Euclidean geometries, but it shares their spirit: abandoning a hypothesis considered universal.

Classical geometry assumes that, at sufficiently small scales, every shape can be approximated by lines and planes. Nature contradicts this hypothesis.

To describe irregular objects, Euclidean metrics are inadequate:

the length of a fractal curve is not a fixed number

the area of a fractal surface can be infinite while being contained in a finite space

the border of a set can itself be a fractal

Precursor models already existed — **Georg Cantor (1845–1918)**, **Giuseppe Peano (1858–1932)**, **David Hilbert (1862–1943)**, **Helge von Koch (1870–1924)** — but they were considered mathematical curiosities. **Mandelbrot** understood that they were **models of reality**.

Nature is not Euclidean

Look at a tree: the trunk divides into main branches, then into secondary branches, then into twigs, then into veins. Look at a lung: bronchi, bronchioles, alveoli. Look at a snowflake, a coastline, a galaxy.

Nature is **branched, irregular, self-similar**. Fractal geometry is the only one capable of describing it with precision.

What it means for those who build models

The distinction between smooth and rough has concrete implications:

in video games, realistic terrains require fractal algorithms

synthetic clouds must use fractal noise

in financial markets, real fluctuations do not follow the smooth Gaussian of classical models

Understanding roughness is the difference between a model that works and one that fails in extreme situations.

The physical boundary of roughness

Mathematics allows for infinite fractals. Physics does not: the world is made of atoms.

The coast is rough down to the scale of sand grains. The lung branches down to the alveoli. Natural roughness always has a physical lower bound.

Yet, some theories — such as loop quantum gravity — suggest that space itself may have a granular or fractal structure at the Planck scale.

If this were so, roughness would not just be a useful model: it would be a fundamental property of reality.

The starting point

Mandelbrot did not invent roughness. Roughness had always been there, in every coastline, in every cloud, in every tree. What he invented was **the way to see it**, and then to **measure it**. He built a mathematical language in which the irregular complexity of the real world was not an anomaly to be ignored, but a property to be quantified.

This change of perspective is the starting point for everything that will follow in this book. In the subsequent chapters we will see:

how roughness is measured with the **dimension of Felix Hausdorff (1868–1942)**

how the most famous fractals are constructed

how to program a computer to generate them

how they appear in the physical, biological, and financial systems that surround us

For now, it is enough to understand one thing: **the world is not Euclidean. It never has been.** And once you begin to see it through the lens of fractal geometry, it is impossible to go back to seeing it as before.

Exercises

1. Coastline Measurement: Take a map of a jagged coastline, for example that of Sardinia or Norway, and try to measure its length first with a 10-centimeter ruler and then with a 2-centimeter one. Note the two results. The difference you obtain is not an error: it is the first empirical demonstration that you are encountering an object with fractal properties. If you want to go deeper, repeat the measurement with a 1-centimeter ruler and plot a graph with the length obtained as a function of the length of the ruler used.

2. Hunting for Roughness: Photograph three natural objects, for example a leaf, a rock, and a cloud. For each, identify at least two different scales of observation and describe how the visible structure changes when passing from one scale to another. Then try to describe the same object using only Euclidean shapes: circles, triangles, cylinders. Note where the Euclidean description fails and where it becomes impossible. This exercise does not require mathematics: it requires observation.

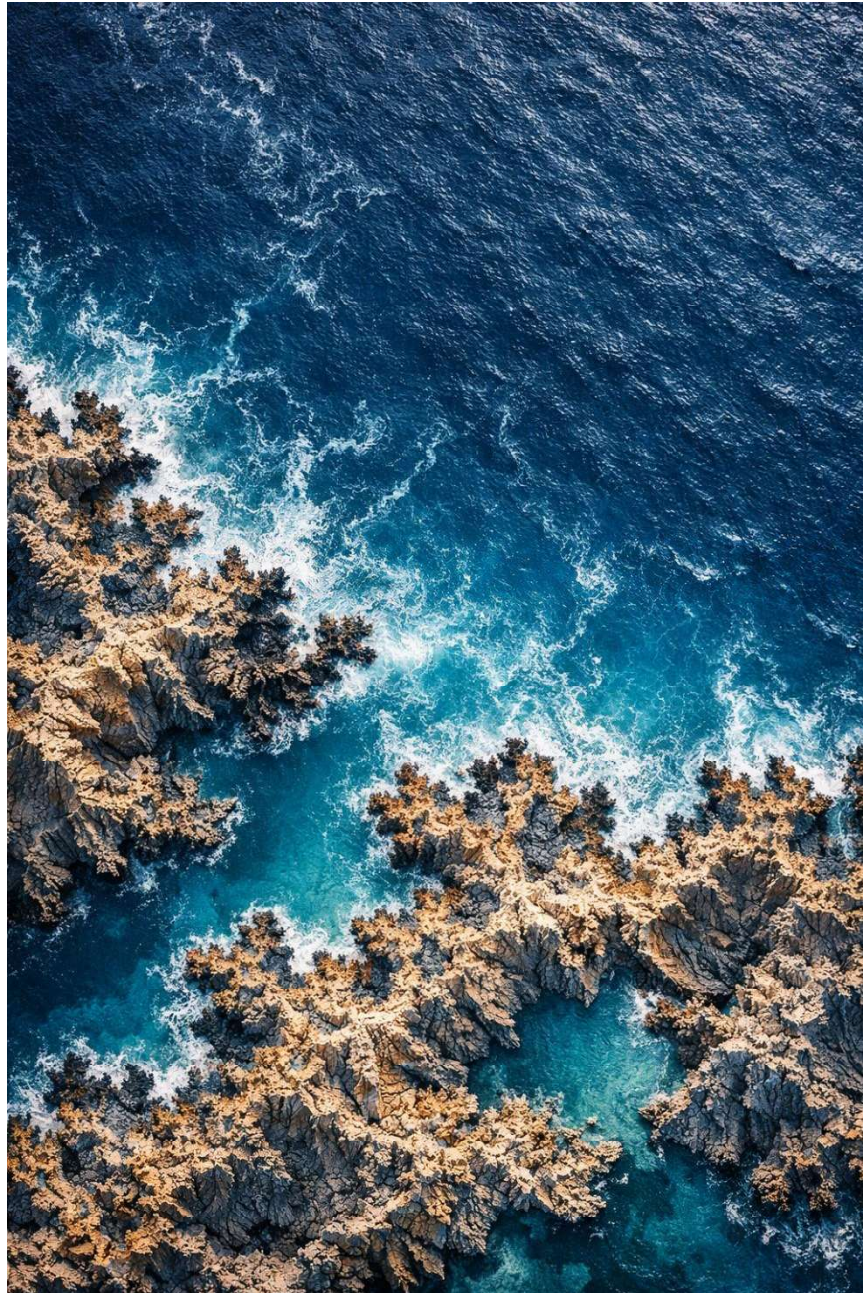


Figure 1 - Imaginary fractal coastline

Image generated with the use of artificial intelligence

Chapter 2 - The Hausdorff Dimension: Measuring Irregularity

“Infinity is not far away: it hides in every detail that repeats itself.”

“Every curve in nature is a signature of eternity.”

In 1918, while Europe sought to pull itself back together after a war that had shattered every certainty, a German mathematician named **Felix Hausdorff (1868–1942)** published a work destined to go almost unnoticed. He did not speak of geopolitics, nor of reconstruction. He did something much more radical: he demonstrated that the **dimension of an object does not need to be an integer**.

For anyone trained in classical geometry, the idea seemed almost absurd. A point has dimension zero. A line has one. A surface has two. A solid has three. These numbers were not the result of a calculation: they were the very way in which space was conceived. To question this structure meant questioning the foundation of reality.

Hausdorff was not denying reality. He was showing that objects exist that are neither lines nor surfaces, but something in between. And that this ‘middle ground’ is not a defect of language, but rather a property measurable with mathematical precision.

The limit of integer dimension

To understand why integer dimension is not enough, let us consider a segment of length 1. Scaled by a factor of 3, it contains exactly three copies of itself. A square scaled by a factor of 3 contains nine copies of the original square. A cube scaled by a factor of 3 contains twenty-seven copies of the original cube.

The pattern is evident: for a line, the number of copies is 3^1 ; for a square, 3^2 ; for a cube, 3^3 . The exponent corresponds to the object’s dimension.

This scheme works perfectly for Euclidean objects. But not for all.

Consider the **Koch curve**, constructed by dividing a segment into three equal parts, replacing the central part with two sides of an equilateral triangle, and repeating the process infinitely. Scaling this curve by a factor of 3, one does not obtain 3 copies nor 9, but 4. The exponent that satisfies $3^D=4$ is neither 1 nor 2: it is approximately **1.26**.

The Koch curve is more than a line, yet not quite a surface. Hausdorff provided the tools to measure exactly this ‘extra amount.’

The formula

For self-similar objects, the Hausdorff dimension takes a surprisingly simple form:

$$D = \log(N) / \log(R)$$

where N is the number of copies and R is the scaling factor.

Applied to the segment: $N=3$, $R=3 \Rightarrow D=1$. Applied to the square: $N=9$, $R=3 \Rightarrow D=2$.
Applied to the Koch curve: $N=4$, $R=3 \Rightarrow D=1.26$.

This formula is not a mathematical trick: it is the operational definition of dimension for objects that do not behave like Euclidean ones. The fractal dimension measures the degree of irregularity of an object. The closer the dimension is to the next higher integer, the more the object tends to fill the space; the closer it is to the lower one, the more rarefied it is.

What it means to be between a line and a surface

What does it mean, physically, to have a dimension of 1.26? A line occupies no area. A surface occupies it completely. An object with dimension 1.26 sits between these two extremes: it does not cover a full area, but it is not so thin as to have zero area. It winds through space in an irregular way, filling some zones and leaving others empty.

The Koch curve is a perfect example. Enlarging any portion, one never sees a smooth line: one always sees the same irregular structure. If you try to cover it with increasingly thin strips, many more are needed than for a straight line, but fewer than for a rectangle. This 'amount of coverage' is what the Hausdorff dimension measures.

The same applies between two and three dimensions. A fractal surface with dimension 2.7 is not a plane, but neither is it a compact solid. It is a surface so convoluted that it almost completely fills the volume in which it is contained. Human lungs are a natural example: the alveolar structure, compressed into a few liters, exceeds **70 square meters** of surface area.

The box counting method

The formula $D = \log(N) / \log(R)$ works for fractals constructed with exact rules. But how do we measure the fractal dimension of a real object, such as a coastline, a neural network, or a microscopic image?

The **box counting method** is used and the number of squares that contain at least part of the object is counted: $N(\epsilon)$. Then the size of the squares is reduced and the process is repeated. If the object is fractal, the ratio between $\log(N(\epsilon))$ and $\log(1/\epsilon)$ converges to a constant value: that is the fractal dimension.

This method can be applied to any digital image and can be implemented with a few lines of code: can give rise to infinite complexity. The applications are vast.

Neural networks and road networks: measuring efficiency with dimension

Fractal dimension is a powerful tool for analyzing complex networks: road, neural, vascular, electrical. All of them branch out to reach the maximum number of points with the minimum path.

A planned American grid city has a fractal dimension close to 2. A medieval European city, grown organically, has a dimension between 1.6 and 1.9. More irregular networks may fall even lower.

These numbers are not just descriptive: they are predictive. A network with high dimension distributes traffic better; one with low dimension creates bottlenecks. Urban planners use fractal dimension as an indicator of efficiency.

Biological neural networks show an analogous behavior. The fractal dimension of the human cortical network is between 1.4 and 1.7, with variations between areas and individuals. Areas with higher dimension tend to be more densely connected and process information in a more parallel manner.

Vascular networks and lungs: when nature optimizes

The human circulatory system is an extraordinary example of fractal optimization. The aorta divides into main arteries, then into secondary arteries, then into arterioles and capillaries. The fractal dimension of the vascular system is about 2.7 in the most vascularized tissues: high enough to ensure that every cell is close to a capillary, but not so high as to completely fill the volume.

The lungs follow the same principle. The bronchial tree maintains an almost constant fractal dimension through all levels of branching, ensuring a uniform distribution of airflow.

Measuring what cannot be seen

The Hausdorff dimension measures something that traditional metrics do not capture: structural complexity.

In tumor tissue, for example, cell edges are more irregular compared to healthy tissue. The fractal dimension of the nuclear border is a useful diagnostic indicator. The same principle applies to EEG signals: a trace from a waking brain has a higher fractal dimension than one in deep sleep or during an epileptic seizure.

Fractal dimension does not replace qualitative analysis: it complements it, providing an objective number where before there was only a subjective assessment.

The maximum dimension: a physical boundary?

In pure mathematics, it is possible to construct sets with any dimension between 0 and 3. Some, such as the **Peano** and **Hilbert** curves, completely fill a surface despite formally being curves.

Physical objects, however, are constrained by real-world limits. A vascular network cannot branch infinitely: at some point, the vessels would become smaller than the molecules they transport. A coastline cannot be irregular at scales smaller than the mineral crystals that compose it.

There is also an upper limit: an object with a dimension too close to 3 would tend to completely fill the space, becoming physically impossible. Nature stops before then.

Hausdorff and his contemporaries

Hausdorff was not just a mathematician: he was also a philosopher and writer. Born in Breslau in 1868, he taught at Bonn and Greifswald. His main work, *Grundzüge der Mengenlehre* (1914), is a pillar of set theory.

The dimension that bears his name was introduced in the article *Dimension und äußeres Maß* (1918). The idea was technically complex, based on Lebesgue measure theory, but the principle was clear: dimension can vary continuously.

Hausdorff never lived to see the impact of his work. A Jew in Germany, he suffered Nazi persecution and in 1942 committed suicide along with his wife and sister-in-law to avoid deportation.

When **Mandelbrot** sought the tools to measure the roughness of nature, he found in the Hausdorff dimension exactly what he needed.

Before Hausdorff, other mathematicians had constructed “pathological” objects: the set of **Georg Cantor**, the curve of **Helge von Koch**, the space-filling curves of **Giuseppe Peano** and **David Hilbert**. They were considered curiosities. Hausdorff provided the theoretical framework that allowed them to be measured.

Limitations and cautions

The Hausdorff dimension is powerful, but requires caution.

The fractal structure of physical objects exists only over a finite range of scales. Box counting applied over too narrow a range can produce misleading values.

Very different objects can have the same fractal dimension: dimension is a synthetic measure and loses information.

The box counting method is sensitive to image quality, the binarization threshold, and the grid position. For this reason, scientific protocols require rigorous repetition.

A number that measures complexity

The Hausdorff dimension has transformed complexity from a vague quality to a measurable quantity. The coastline of Great Britain has a dimension of about 1.25. The coastline of Norway about 1.52. These numbers predict how the measured length changes with scale.

Fractal dimension is not the only measure of complexity, but it remains the most fundamental, but it is the most fundamental. It is the cornerstone of Mandelbrot's geometry of nature.

In the next chapter, we will explore **self-similarity**, the property that links the repeating structure to the number that measures it.

For now, it is enough to understand that dimension is no longer an obvious property. It is something that is calculated, measured, and varies continuously between integers. And it is the quantitative signature of the complexity of the real world.

Exercises

1. Calculating Dimension: Using the formula $D = \log(N)/\log(R)$, calculate the Hausdorff dimension of a fractal that divides into 8 parts with a scaling ratio of 3. Hint: substitute $N = 8$ and $R = 3$ into the formula and calculate the logarithm in any base, as long as it is the same for the numerator and denominator. The result is a simple rational number.

2. Manual Box-Counting: Overlay a square grid on the image of a fern leaf and count how many squares are occupied. Halve the grid size and repeat to estimate the fractal dimension. If you have access to graph paper and a printed photograph, you can do this exercise without any software: the result will not be as precise as that of an algorithm, but it will give you a concrete idea of how the method works.

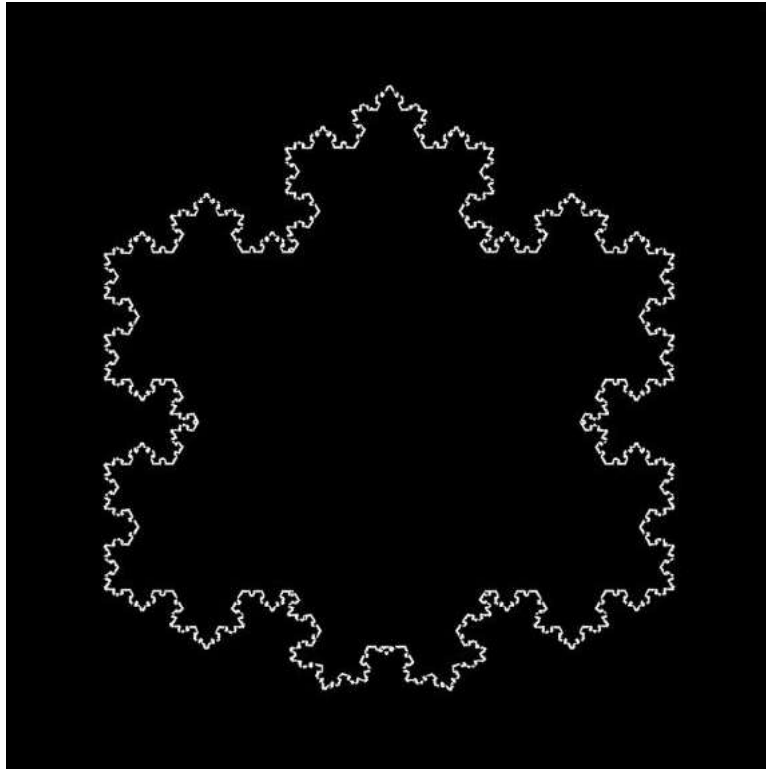


Figure 2 - The Koch curve

Image generated with the aid of artificial intelligence

Chapter 3 - Self-Similarity: The Mirror of Infinity

«The algorithm does not create the future: it measures it, one step at a time.»

«AI is the lens that invites us to look at the infinite with human eyes.»

There is a precise moment in the history of science when an apparently trivial observation transforms into a revolution. That moment does not always have the dramatic appearance we would expect: no falling apple, no "eureka" shouted in a bathtub. Sometimes it is simply a researcher staring at a noisy graph who wonders, with the methodical calm of one who does not trust coincidences, why that noise always resembles itself. Benoît Mandelbrot, in the early sixties, was working for IBM when he found himself analyzing transmission data from a transatlantic telephone line. The problem was practical: transmission errors were irregular and unpredictable. Mandelbrot observed the data on different time scales: an hour, a minute, a second. The structure remained the same at every scale. Periods of silence interrupted by bursts of errors, bursts that contained within them shorter periods of silence, interrupted by even shorter bursts. The pattern repeated, unchanged, at every scale of observation. It was not random noise. It was something far stranger — and far more orderly: it was self-similarity.

The discovery no one had looked for

Self-similarity was not a new concept. Mathematicians of the late nineteenth century had already constructed objects endowed with this property, such as the Helge von Koch curve and the Georg Cantor set, classifying them as mathematical curiosities or pathologies: limiting cases useful for testing definitions, not models of something real. No one had yet realized that this property might be the rule rather than the exception, not the exception, in the structure of the physical world. Mandelbrot understood something different. He saw that the same structure appearing in electrical noise also appeared in the data on cotton prices he had studied years earlier, in the fluctuations of the Nile level analyzed by the hydrologist Harold Hurst, in the distribution of word lengths in written texts. The deep link between these phenomena did not lie in a common physical force, but in the presence of complex dynamic processes driven by a long-term memory: every variation preserved a statistical trace of previous variations, cascading across different scales. Wherever he looked, Mandelbrot found the same signature: structures that repeated at different scales, where the behavior of the whole was mirrored in the behavior of every part. This observation had enormous implications. If nature repeatedly uses the same pattern on different scales, then describing a natural object does not require listing every detail at every scale: it is enough to describe the pattern and specify how it replicates. A forest is not the sum of millions of individual leaves; it is a set of branching structures that all follow the same growth logic, repeated from the trunk to the main branches, from the branches to the twigs, from the twigs to the petioles. Every part resembles the whole: this is the essence of self-similarity.